Chiral Transition and Some Issues on the Scalar Mesons

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Some issues on the low-mass scalar mesons are discussed in relation to the chiral transition of QCD vacuum. The importance to explore the possible collective nature of the σ meson is emphasized in association with the chiral properties of nuclear media.

INTRODUCTION

There are serious controversies on the nature of the low-lying scalar mesons with a mass lower than 1 GeV, especially the σ meson. In the non-relativistic constituent quark model, $J^{PC} = 0^{++}$ is realized as a ${}^3\mathrm{P}_0$ state, which implies that the mass of the σ should be in the 1.2-1.6 GeV region. Therefore some mechanism is needed to down the mass. The possible mechanisms so far proposed include:
(1) The color magnetic interaction between the di-quarks as advocated by Jaffe[1].
(2) The collectiveness of the scalar mode as the pseudoscalar mode; a superposition of $q\bar{q}$ states, which collectiveness is due to chiral symmetry[2].
(3) The σ meson may be a molecular states of the NG bosons naively suggested by the fact that the unitarized chiral dynamics could account for the existence of the σ pole[3].
In this talk, I shall give some arguments to adovocate the viewpoint (2) above.

The basic idea underlying my talk is that the low-energy hadron physics may be regarded as a study of the nature of QCD vacuum and hopefully its symmetry properties: In other words, hadron physics is a condensed matter physics of the QCD vacuum. In fact, the QCD vacuum is realized non-perturbatively and hadrons are elementary excitations on top of the non-perturbative vacuum, while QCD itself is written solely in terms of quark and gluon fields. Moreover, symmetries in (classical) QCD Lagrangian are not manifest; color SU(3) is not manifest owing to the confinement and (approximate) chiral symmetry is spontaneously broken.

Such a viewpoint as described above on the vacuum in quantum field theories was introduced by Nambu[2, 4]. In the paper[5] entitled "Quasiparticles and Gauge invariance in the Theory of Superconductivity", Nambu showed that the appearance of the would-be massless collective mode in the broken phase is a logical consequence of the gauge invariance of the theory; here we remark that the appearance of a massless mode which couples to the longitudinal part of the electro-magnetic current and hence insures the gauge invariant explanation of Meissner effect had been shown earlier by Bogoliubov[6] and Anderson[7] and others[8]. Then in an important but relatively less known paper[4] entitled " Axial Vector Cur-

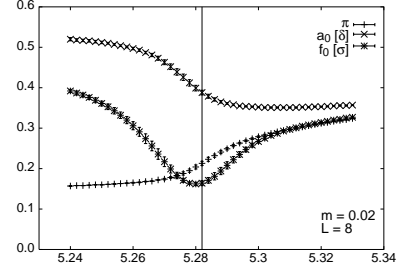

FIG. 1: The lattice calculation [10] of the temperature dependence of the generalized masses as defined by $m_\sigma^2 = \chi_\sigma^{-1}$ for the σ meson where $\quad \chi_\sigma = \langle(\bar{q}q)^2\rangle$.

rent Conservation in Weak Interactions", he suggested that the fundamental theory of the hadron dynamics could be obtained by replacing the gauge invariance, the energy gap, and the collective excitation in the theory of superconductivity with γ_5 invariance (chiral symmetry), baryon mass m_f, and the pion: The pion automatically emerges as a bound state of baryon pairs; the nonzero meson masses would indicate that the chiral symmetry is not rigorous. This scenario was shown to be the case in a model calculation by Nambu and Jona-Lasinio [2] in the famous paper entitled "Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity": It is noteworthy here that a isoscalar-scalar meson with $J^{PC} = 0^{++}$ emerges with the mass $2m_f$ as another collective mode as the pion does; the scalar meson is called the σ meson, which is named f_0 by the PDG.

Now the σ meson has another important aspect. First notice that the chiral transition is a phase transition of QCD vacuum with $\langle\bar{q}q\rangle$ being the order parameter, as clearly shown by recent lattice simulation of QCD[9, 10].

If a phase transition is of 2nd order or *weak* 1st order, there exists "soft" modes which decreases it mass when the system approach the critical point; the soft modes are actually fluctuations of the order parameter of the phase transition[11]. For chiral transition, the relevant fluctuation is described by $\langle(: \bar{q}q :)^2\rangle$, which has the same quantum numbers as the σ-meson does, i.e., $(I = 0, J^{PC} = 0^{++})$. Accordingly the σ meson can become the soft mode of chiral transition at $T \neq 0$ and/or $\rho_B \neq 0$[12]: $m_\sigma \to 0$, $\Gamma_\sigma \to 0$ A lattice calculation of the generalized masses [10] shows a behavior consistent with the soft mode nature of the σ meson, as shown in Fig.1:
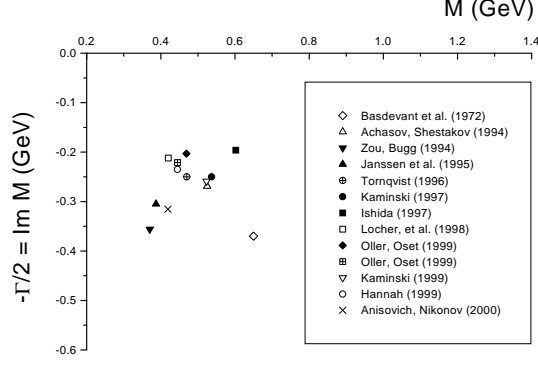
FIG. 2: The poles of the S-matrix in the complex mass plane (GeV) for the σ meson; complied in [17].

FIG. 3: The π-π phase shift in the σ channel calculated by means of N/D method [19] reproduces the experimental data with the incorporation of the σ pole in the s channel as well as the ρ pole in the t channel; the phase shift obtained without the σ pole fails in reproducing the empirical data.

One sees (1) the softening of σ (2) a degeneracy of the σ and π at high T. Moreover one may notice that $U_A(1)$ symmetry is not restored even at high T; $m_\delta \leftrightarrow m_\pi$.

But, what is the significance of the σ in hadron physics[13, 14]?

a) A pole in the σ channel in the low-mass range is observed in the π-π scattering matrix[15, 16]. As a compilation of the pole positions of the σ obtained in the modern analyses, see [17]. It is recognized that respecting chiral symmetry, unitarity and crossing symmetry is essential to reproduce the phase shifts both in the σ (s)- and ρ (t)-channels with a low-mass σ pole[18, 19].

b) Moreover, the σ is also seen in decay processes from heavy particles; $D^+ \to \pi^- \pi^+ \pi^+$ [21].

c) As is well known[22], the σ with a mass $400 \sim 600$ MeV is responsible for the intermediate range attraction in the nuclear force; without the σ contribution, any nucleus can not be bound, nor possible our existence.

d) The σ can accounts for $\Delta I = 1/2$ enhancement in $K^0 \to 2\pi$ compared with $K^+ \to \pi^+ \pi^-$[23].

e) The empiricalvalue of the π-N sigma term $\Sigma_{\pi N} \sim$ 40-50 MeV may be accounted for by the collectiveness of the σ [24], as will be discussed shortly.

The quark content in the scalar channel of a nucleon, $\langle N|\bar{q}q|N \rangle$ can be enhanced by the collective σ mode so much as to reproduce the empirical value of the π-N sigma term[24]. In fact, using the quark contents given in Table 1, we have

$$
\begin{aligned}
\Sigma_{\pi N} &= \hat{m}\langle \bar{u}u + \bar{d}d \rangle_N \\
&= 5.5\text{MeV} \times (4.97 + 4) \simeq 50\,\text{MeV}, \quad (1)
\end{aligned}
$$

which is similar to the empirical value in contrast to the value given by the naive quark model $5.5 \times (2+1) \simeq 17$ MeV.
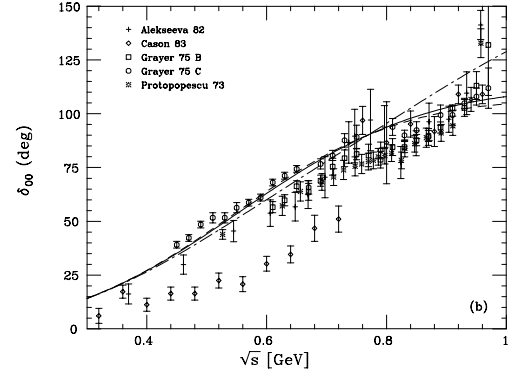
TABLE I: Light quark contents of baryons calculated with a chiral quark model with the σ meson cloud incorporated [24]. The numbers in ( ) are those in the naive quark model.

B	$\langle \bar{u}u \rangle_B$	$\langle \bar{d}d \rangle_B$	$\langle \bar{s}s \rangle_B$
P (938)	4.97 (2)	4.00 (1)	0.53 (0)
Λ^0 (1115)	3.63(1)	3.63(1)	1.74(1)
Δ^{++} (1232)	3.66(2)	0.76 (0)	0.26 (0)
Ω^- (1672)	0.72 (0)	0.72 (0)	3.71 (3)

IS THE OBSERVED σ MESON RELATED WITH THE CHIRAL RESTORATION?

Is the pole observed in the π-π phase shift really the σ as the quantum fluctuation of the order parameter of the chiral transition? A change of the environment will lead to that of the modes coupled to the order parameter. Thus production of the σ-meson in nuclear medium should be useful for exploring the existence of the σ and the possible restoration of chiral symmetry at finite density [13, 25, 26, 27].

One may ask what good observables is to see the softening in the σ channel in nuclear medium. A particle might loose its identity when put in a medium, by various process like $\sigma \leftrightarrow 2\pi$, $\sigma \leftrightarrow$ p-h, π+p-h, Δ-h, $\pi + \Delta$-h ... Thus one needs to calculate the strength function of the hadron channel in the media[12, 28].

The surprise was, such an enhancement had been seen by an Experiment by CHAOS collaboration[29] at $T = 0$ but at $\rho_B \neq 0$ by the reaction $A(\pi^+, \pi^+\pi^\pm)A'$, where the atomic number A runs from 2 to 208; we remark that the experiment was motivated by other purpose [30] but not by exploring partial restoration of chiral symmetry in a nuclear medium. Clearer information on the strength function in a nuclear medium may be provided with electro-magnetic probes; the reaction $A(\gamma, \pi^0\pi^0)A'$ performed by TAPS collaboration[31] clearly shows a
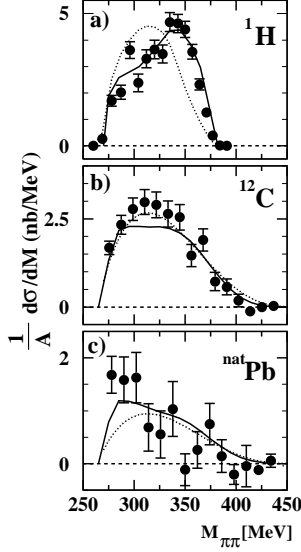
FIG. 4: Photo-2π cross section on nuclear targets[31]. The incident energy belongs to the region 400 to 600 MeV.

softening of the cross section, i.e., a downward shift of the cross section as a function of the invariant energy of 2 pions, as shown in Fig.4.

A calculation of the strength function in the σ channel in a nuclear medium with $\rho_B \neq 0$ was made by Hatsuda et al [32] based on a linear σ model; they showed that an enhancement of the strength function near $2m_\pi$ threshold as observed in [29, 31] might be accounted for in terms of a partial restoration of chiral symmetry in heavy nuclei. Interestingly enough, Jido et al [33] showed that a similar enhancement can be obtained even in the nonlinear realization of chiral symmetry; the key ingredient is the wave-function renormalization of the pion field in the medium. Subsequently, the N/D method [20] a la Igi-Hikasa[19]is applied to examine the strength function in the scalar and vector channels by Yokokawa et al[34]. They found a softening of the spectral function both in the σ and the ρ meson channels being associated with partial restoration of chiral symmetry in the nuclear and hadron media: Fig.5 shows the softening of the spectral function in the σ channel; following [33], the medium effect of chiral properties was taken into account by the density-dependent pion decay constant $f_\pi^*(\rho)$ which is closely related with the wave-function renormalization mentioned above. In Fig.6, shown is the softening of the σ meson pole in the 2nd Riemann sheet[34].
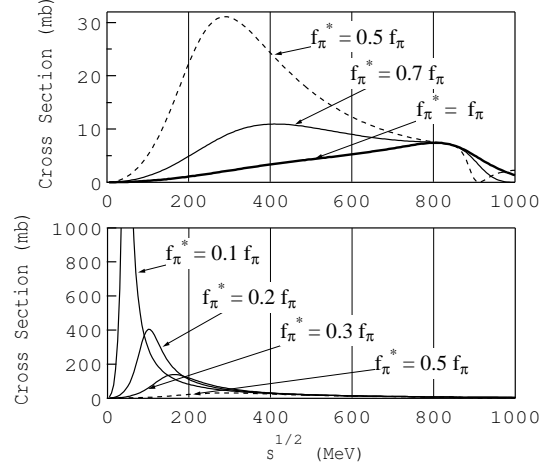

FIG. 5: The T matrix in the N/D method. The in-medium π-π cross sections in $I=J=0$ channel. The upper (lower) panel shows the case of small (large) restoration corresponding to $0.5f_\pi < f_\pi^* < f_\pi$ ($0.1f_\pi < f_\pi^* < 0.5f_\pi$). Taken from [34].
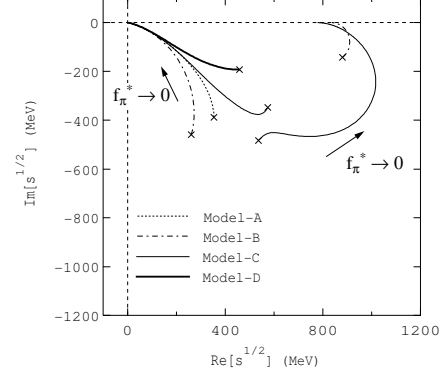

FIG. 6: The movement of the poles in $I=J=0$ channel along with the the decrease of f_π^*. The crosses are the pole positions in the vacuum. Taken from [34].

SUMMARY

Although there have been accumulation of experimental evidence of the σ pole with a low mass 500 to 600 MeV in the pi-pi scattering matrix, there are serious controversies on the nature of the σ meson: Naive quark model is in trouble for explaining such a low-mass state in the $^3P_0$ state; it may be a four-quark or a collective q-q̄ or π-π molecular state. The σ as a collective q-q̄ state is identified as the quantum fluctuation of the order parameter of the chiral transition. The existence of such a collective mode in the scalar channel can account for some phenomena in hadron physics which otherwise remain mysterious.

To explore the nature of the σ meson, trying to create a σ-mesonic mode in a nuclear medium may be helpful;

a peculiar enhancement of the spectral function in the σ channel in the lowering energy side may be observed along with a partial restoration of chiral symmetry in the medium. Such an enhancement might have been already observed in some experiments.

Recently, possible N_c-dependence of the nature of the σ meson has been noticed by some authors[35, 36]; T. Schaefer showed that at $N_c = 3$ the low mass σ exists which is described as a linear combination of q-q̄ and $(q\bar{q})^2$. However, for larger N_c, m_σ goes up and the σ becomes mainly composed of q-q̄. The same problem is examined by others but somewhat different conclusions are deduced[36]. In passing, we remark that a lattice simulation with dynamical fermions shows a low-mass σ degenerate with the ρ meson[37].

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